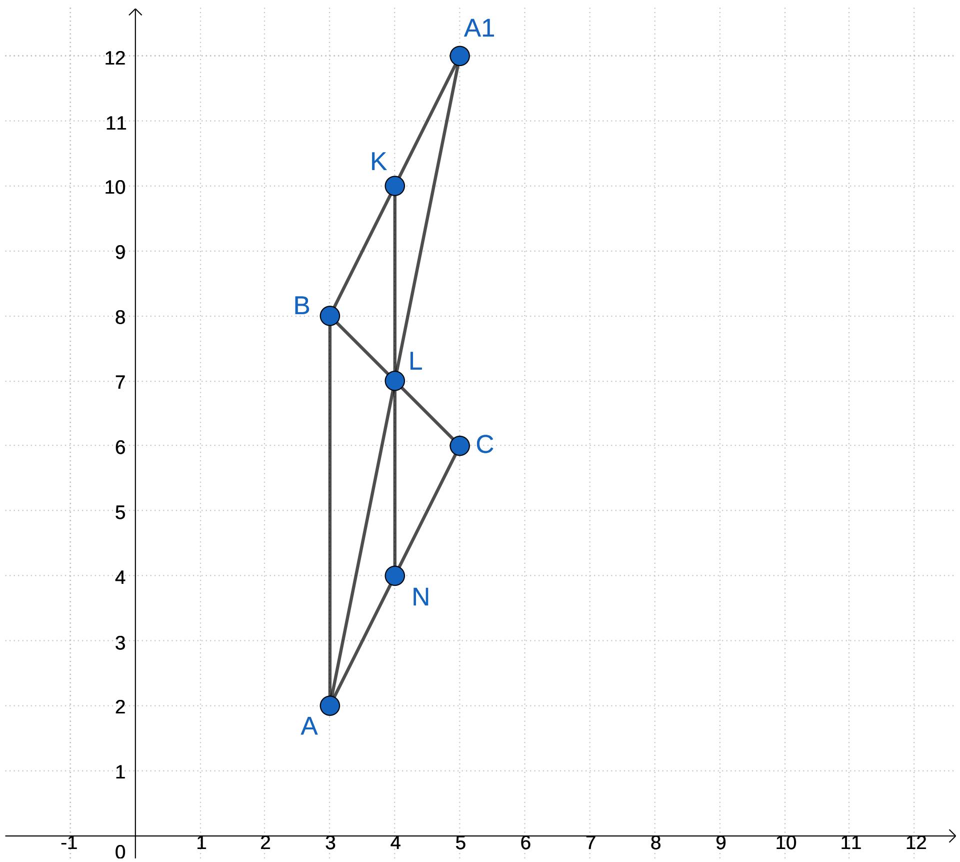
Demo Problem 2

1 Problem

ABC is a triangle. Points N and L are the midpoints of the sides [A,C] and [B,C] respectively. Point A1 is symmetric to point A with respect to point L. K is the midpoint of segment [A1,B]. Prove that the points N, L, and K are collinear.

2 Diagram



3 Input to the Program

3.1 Premises

Euclidean Description of the Problem
Triangle ABC.
N is the midpoint of [A,C].
L is the midpoint of [B,C].
A1 is symmetric to A with respect to L.
K is the midpoint of [A1,B].

Cartesian Description of the Diagram
Point A has coordinates (3,2).
Point B has coordinates (3,8).
Point C has coordinates (5,6).
Point N has coordinates (4,4).
Point L has coordinates (4,7).
Point A1 has coordinates (5,12).
Point K has coordinates (4,10).

3.2 Goal

Prove that points N, L, and K are collinear.

4 Notes

- In this particular example, there was no need to specify the lines (A,L,A1), (B,K,A1), (A,N,C), and (N,L,K) because the program is able to derive these lines automatically through symmetry and midpoint premises. However, in the general case it is advised to included all the lines in the diagram except when of course the linearity of a group points is not known.

5 Proof

Start

We will prove that points N, L, and K are collinear by showing that they form a straight angle [K,L,N]

We show that the size of angle [K,L,N] is 180 degrees

Start

The size of angle [K,L,N] will be computed as the sum of the sizes of sub-angles [B,L,K] and [B,L,N]

We find the combined size of angles [B,L,K] and [B,L,N]

Start

The combined size of angles [B,L,K] and [B,L,N] will be computed by first proving that angles [B,L,K] and [B,L,N] are supplementary and then deducing that their combined size is 180

1. We prove that angles [B,L,K] and [B,L,N] are supplementary

Start

Angles [B,L,K] and [B,L,N] will be proved supplementary by showing they are equal to angles that are supplementary

1. We prove that angles [B,L,K] and [A,B,C] are equal

Start

Angle equality of [B,L,K] and [A,B,C] will be proved by showing they are alternate interior angles formed by parallel lines (K,L) and (A,B) and their intersector (B,C)

We shall prove that lines (K,L) and (A,B) are parallel

Start

Lines (K,L) and (A,B) will be proved parallel using Thales theorem

In the triangle AA1B we have

1. Point L is to be proved the midpoint of segment [A,A1]

Start

Point A1 is symmetric to point A with respect to point L

End

2. Point K is to be proved the midpoint of segment [A1,B]

Start

It is given in the premises that point K is the midpoint of segment [A1,B]

End

Therefore, lines (K,L) and (A,B) are parallel

End

Therefore, angles [B,L,K] and [A,B,C] are equal

End

2. We prove that angles [B,L,N] and [B,L,N] are equal

Start

An angle equals itself

End

3. We prove that angles [A,B,C] and [B,L,N] are supplementary

Start

Angles [A,B,C] and [B,L,N] will be proved supplementary by showing they are formed by parallel lines (A,B) and (L,N) and are on the same side of their intersector (B,C)

We shall prove that lines (A,B) and (L,N) are parallel

Start

Lines (A,B) and (L,N) will be proved parallel using Thales theorem

In the triangle ABC we have
1. Point N is to be proved the midpoint of segment [A,C]
Start
It is given in the premises that point N is the midpoint of segment [A,C]
End
2. Point L is to be proved the midpoint of segment [B,C]
Start
It is given in the premises that point L is the midpoint of segment [B,C]
End
Therefore, lines (A,B) and (L,N) are parallel
End
Therefore, angles [A,B,C] and [B,L,N] are supplementary
End
Therefore: angles [B,L,K] and [B,L,N] are supplementary
End
2. We deduce that – since angles [B,L,K] and [B,L,N] are supplementary – their combined size is 180
End
Therefore, the size of angle [K,L,N] is size=180
End
Therefore, points N, L, and K are collinear
End