Demo Problem 1
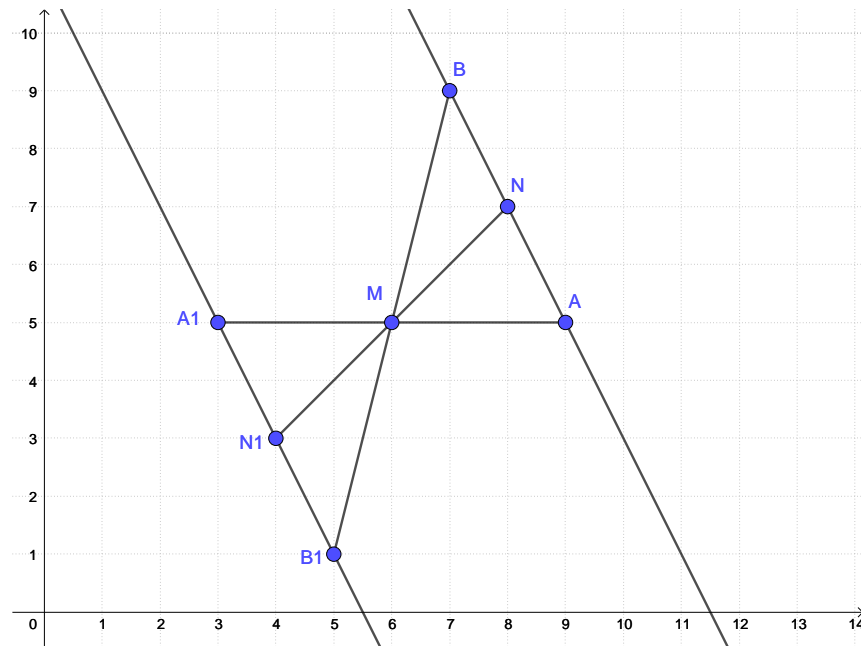
1   Problem

M, A, and B are three non-collinear points. N is the midpoint of segment [A,B]. Points A1, B1, and N1 are respectively symmetric to points A, B, and N with respect to point M. Prove that lines (A,B) and (A1,B1) are parallel.

2   Diagram

3 Input to the Program

3.1 Premises

Euclidean Description of the Problem	Cartesian Description of the Diagram
N is the midpoint of [A,B].	Point A has coordinates (9,5).
Point A1 is symmetric to point A with respect to point M.	Point B has coordinates (7,9).
Point B1 is symmetric to point B with respect to point M.	Point M has coordinates (6,5).
Point C1 is symmetric to point C with respect to point M.	Point N has coordinates (8,7).

3.2 Goal

Prove that lines (A,B) and (A1,B1) are parallel.

4 Notes

- In this particular example, there was no need to specify the lines (A,N,B), (A,M,A1), (B,M,B1), and (N,M,N1) because the program is able to derive these lines automatically through symmetry and midpoint premises. However, in the general case it is advised to included all the lines in the diagram except when of course the linearity of a group points is not known. For example, in this problem; we cannot directly imply that points A1, N1, and B1 are collinear and hence line (A1,N1,B1) cannot be added to the problem premises.

5 Proof

Start
| The two lines (A1,B1) and (A,B) will be proved parallel using information about existing alternate interior angles [B1,A1,A] and [A1,A,B]
| We will prove that angles [B1,A1,A] and [A1,A,B] are equal
| Start
| | Angle [A,A1,B1] is the same as angle [B1,A1,M]
| | Angle [A1,A,B] is the same as angle [B,A,M]
| | Angles [B1,A1,M] and [B,A,M] will be proved equal
| | Start
| | | Angle equality of [B1,A1,M] and [B,A,M] will be proved using SAS triangle congruence
| | | The two triangles A1B1M and ABM have respectively:
| | | 1. We prove that segments [A1,M] and [A,M] are equal
| | | Start
| | | | Point A1 is symmetric to point A with respect to point M
| | | End
| | | 2. We prove that segments [B1,M] and [B,M] are equal
| | | Start
| | | | Point B1 is symmetric to point B with respect to point M
| | | End
| | | 3. We prove that angles [A1,M,B1] and [A,M,B] are equal
| | | Start
| | | | Angles [A1,M,B1] and [A,M,B] are equal because they are vertically-opposite angles
| | | End
| | | Therefore:
| | | Triangles A1B1M and ABM are congruent, and hence: angles [B1,A1,M] and [B,A,M] are equal
| | End
| | Therefore, angles [A,A1,B1] and [A1,A,B] are equal
| End
| Angles [B1,A1,A] and [A1,A,B] are equal, and hence lines (A1,B1) and (A,B) are parallel
End